

Matrices are a rectangular array of numbers, algebraic symbols, or functions with a defined set of operations.

$$\begin{array}{c}
 \text{column 1} \quad \text{column 2} \quad \text{column 3} \quad \dots \\
 \text{row 1} \left[ \begin{array}{cccc} a_{1,1} & a_{1,2} & a_{1,3} & \dots \end{array} \right] \\
 \text{row 2} \left[ \begin{array}{cccc} a_{2,1} & a_{2,2} & a_{2,3} & \dots \end{array} \right] \\
 \text{row 3} \left[ \begin{array}{cccc} a_{3,1} & a_{3,2} & a_{3,3} & \dots \end{array} \right] \\
 \vdots \left[ \begin{array}{cccc} \vdots & \vdots & \vdots & \ddots \end{array} \right]
 \end{array}$$

The **dimensions** of a matrix is defined by the number of Rows and then, the number of Columns (R x C).

1. What are the dimensions of each the following matrices?

a.  $\begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & 5 \end{bmatrix}$

b.  $[2 \quad -1 \quad 3 \quad 7]$

c.  $\begin{bmatrix} 4 & x & 3 & 2y \\ 3z & 1 & 0 & -2 \end{bmatrix}$




2. Use the following matrices to determine the value of the suggested elemental expressions.

$A = \begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$

$B = \begin{bmatrix} -5 & 2 \\ -7 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -2 & 6 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 0 \\ -4 & 3x \\ 3 & -2 \end{bmatrix}$

$E = \begin{bmatrix} 5 & x & 3 \\ 0 & 7 & y+1 \end{bmatrix}$

a.  $A_{2,1} =$

b.  $C_{2,2} + D_{3,1} - B_{2,1} =$

c.  $D_{2,2} + E_{1,2} - E_{2,3} =$




In the **Transpose, (T)**, of a matrix the columns and rows are flipped & each element changes from  $a_{r,c}$  to  $a_{c,r}$ .

3. Find the transpose of each of the matrices shown below (Commonly, a superscript or exponent capital *T* is used to denote the transpose of a matrix. However, sometimes an apostrophe is also used.)

a.  $\begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix}^T =$

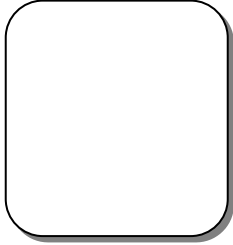
b. Given,  $[A] = \begin{bmatrix} 5 & 2x & 3 \\ 1 & 0 & 4 \end{bmatrix}$  determine  $[A]^T$

There are two types of **special matrices** that are commonly used an Identity matrix and a Zero matrix.

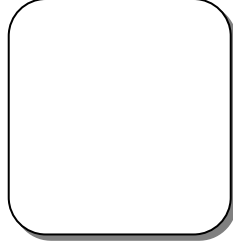
- **Identity Matrices** are only square matrices that have only 1's on their main diagonal and 0's for the remaining elements.
- **Zero Matrices** can be any dimension and they only contain 0's for every element.

4. Create the requested special matrices.

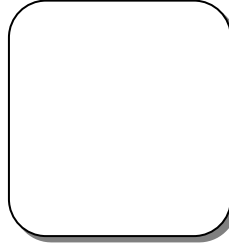
a.  $I_2$



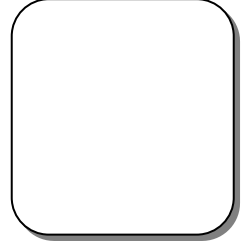
b.  $I_4$



c.  $O_{3 \times 2}$



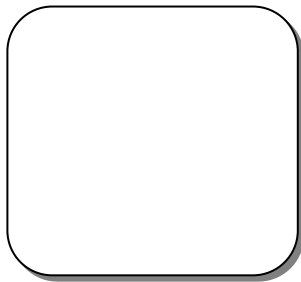
d.  $O_{1,4}$



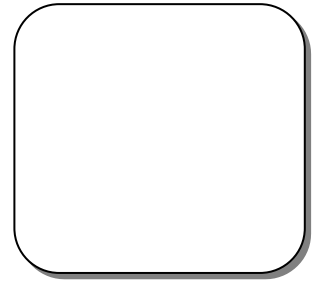
**Scalar Multiplication.** A coefficient in front of matrix implies multiplication of each element by that coefficient.

5. Simplify each of the following matrices by 'distributing' the scalar.

a.  $-2 \cdot \begin{bmatrix} 3 & 0 & -1 \\ -2 & 5 & 4 \end{bmatrix} =$



b. Given,  $[A] = \begin{bmatrix} y & -1 \\ 0 & 3x \end{bmatrix}$ , determine  $x \cdot [A]$



**Matrix Addition.** The sum of matrices can only be found for matrices that have identical dimensions and then each element in the same location is added to one another.

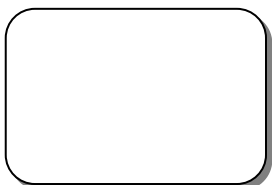
6. Find the following matrix sums if possible.

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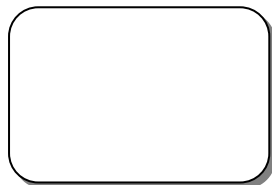

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 2x & 1 \\ 0 & 3 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 1 \\ -2 & 3x \\ 3 & -2 \end{bmatrix} \quad E = \begin{bmatrix} 5 & x & 3 \\ 0 & 7 & y+1 \end{bmatrix}$$


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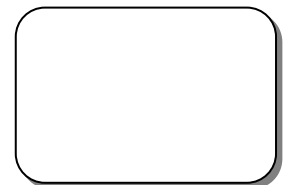
a.  $[A] + 2[B] =$



b.  $3[C] + 2[E] =$



c.  $[B] + [B]^T =$



7. Find the following matrix sums if possible.

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 2x & 1 \\ 0 & 3 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 1 \\ -2 & 3x \\ 3 & -2 \end{bmatrix} \quad E = \begin{bmatrix} 5 & x & 3 \\ 0 & 7 & y+1 \end{bmatrix}$$

a.  $2[C] + [D] =$

b.  $-2[A] + 3[I_2] =$

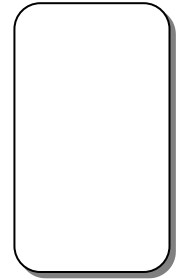
c.  $[0_{2 \times 3}] + [D]^T =$



8. In each of the matrix equations, solve for the unknown variables.

a.  $\begin{bmatrix} x & 5 \\ 3 & 2y \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 2 & 11 \end{bmatrix}$

b.  $2 \begin{bmatrix} 2 & a \\ 3 & 2c \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ b & -6 \end{bmatrix} = \begin{bmatrix} 9 & 11 \\ 2 & a \end{bmatrix}$



**Matrix Multiplication.** The product of matrices is NOT commutative and can only be determined if the inner dimension align (e.g. Matrix  $[A]$  has the dimensions  $m \times n$ , the product of  $[A][B]$  can only be found if matrix  $[B]$  has the dimensions  $n \times p$ . The columns of matrix  $[A]$  must match the rows of matrix  $[B]$ . Furthermore, their product will have the dimensions  $m \times p$ )

9. Determine whether or not the following products are possible & if so give the dimensions of the matrix product.

- Matrix  $[A]$  has dimensions  $3 \times 3$
- Matrix  $[B]$  has dimensions  $2 \times 3$
- Matrix  $[C]$  has dimensions  $4 \times 2$
- Matrix  $[D]$  has dimensions  $2 \times 2$

a. If possible, what are the dimensions of  $[B] \cdot [A]$ ?

d. If possible, what are the dimensions of  $[D] \cdot [C]^T$ ?

b. If possible, what are the dimensions of  $[A] \cdot [B]$ ?

e. If possible, what are the dimensions of  $[B]^T \cdot [B]$ ?

c. If possible, what are the dimensions of  $[B] \cdot [B]$ ?

f. If possible, what are the dimensions of  $[A]^2$ ?

## Matrix Multiplication

10. Find the following matrix products if possible.

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 4 \\ 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -3 & 1 \\ 4 & 2 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 1 \\ -2 & 5 \\ 3 & -2 \end{bmatrix}$$

$$E = \begin{bmatrix} x & 3 \\ 5 & 2y \end{bmatrix}$$

a.  $[A] \cdot [B] =$

b.  $[B] \cdot [A] =$

c.  $[D] \cdot [C] =$

d.  $[B] \cdot [E] =$

e.  $[A] \cdot [I_2] =$

f.  $[C] \cdot [C]^T =$

11. In each of the matrix equations, solve for the unknown variables.

a.  $\begin{bmatrix} x & 3 \\ 1 & 2y \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & a \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2b & 1 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} c & 7 \\ 10 & 8 \end{bmatrix}$